Available online at www.sciencedirect.com
ScienceDirect
International Journal of Multiphase Flow 33 (2007) 575–594
International Journal of Multiphase Flow
www.elsevier.com/locate/ijmulflow

Pressure recovery of two-phase flow across sudden expansions

Wael H. Ahmed, Chan Y. Ching *, Mamdouh Shoukri

Department of Mechanical Engineering, McMaster University, Hamilton, Ont., Canada L8S 4L7

Received 20 April 2006; received in revised form 12 December 2006

Abstract

An analytical formulation for the pressure recovery of two-phase flow across a sudden expansion was developed. The formulation takes into account the change in void fraction across the expansion, the pressure difference between the upstream flow and the downstream face of the expansion and the additional wall shear stress in the developing region downstream of the expansion. Experiments were performed using air–oil two-phase flow to evaluate the relative contribution of the different terms to the pressure recovery for three area ratios of 0.0625, 0.25 and 0.444. The current formulation, which takes into account all the relevant terms, was found to improve the prediction of the pressure recovery over existing models.

Keywords: Pressure recovery; Air–oil two-phase flow; Sudden area expansion

1. Introduction

Gas–liquid two-phase flows are important in an increasing number of applications and industries, including the nuclear energy and oil and gas industries. In these applications, the two-phase flow can occur in complex piping geometries that include sudden area changes, orifices, bends and valves under a variety of different flow conditions. In order to properly design such two-phase flow systems, it is important to understand the effect of these piping components on the two-phase flow parameters such as pressure, void fraction and flow pattern. When frequent area changes are present in piping systems, the losses caused by these become significant and their accurate prediction is important to evaluate the system performance. While there have been a large number of studies on gas–liquid flows in constant area pipes, there are fewer studies on two-phase flow across sudden area expansions.

Lottes (1960), Mendler (1966), Janssen and Kervinen (1966) and McGee (1966) were among the first to investigate two-phase flow through sudden expansions. In the majority of these studies, the void fraction and pressure changes were used to characterize the flow across the sudden area expansion. The pressure recovery across a sudden expansion is usually obtained by applying the conservation equations to a control volume surrounding the sudden expansion (Richardson, 1958; Lottes, 1960; McGee, 1966; Delhaye, 1981; Attou and

* Corresponding author. Tel.: +1 905 525 9140x24998; fax: +1 905 572 7944.
E-mail address: chingcy@mcmaster.ca (C.Y. Ching).

0301-9322/$ - see front matter
doi:10.1016/j.ijmultiphaseflow.2006.12.005

Table 1
Models for pressure recovery across a sudden expansion

Romie model	$P_2 - P_1 = \frac{1}{2}\rho_L V_0^2(2\sigma)\left[x^2\left(\frac{\rho_L}{\rho_G}\right)\left(\frac{1}{\alpha_1} - \frac{\sigma}{\alpha_2}\right) + (1-x)^2\left(\frac{1}{1-\alpha_1} - \frac{\sigma}{1-\alpha_2}\right)\right]$
Lottes (1960)	$P_2 - P_1 = \frac{1}{2}\rho_L V_0^2(2\sigma)\cdot\left[\frac{1}{(1-\alpha_1)(1-\alpha_2)} - \sigma\left(\frac{1}{1-\alpha_2}\right)^2\right]$
Richardson (1958)	$P_2 - P_1 = \frac{\rho_L V_0^2(1-x)^2(1-\sigma^2)\sigma}{2\cdot(1-\alpha)}$
Mendler et al. (1961)	$P_2 - P_1 = \frac{\rho_L V_0^2}{2g}\left[1 + x\left(\frac{v_{LG}}{v_L}\right)\right]\cdot 2\sigma(1-\sigma)$
Delhaye (1981)	$P_2 - P_1 = \sigma(1-\sigma)G_1^2\left[\frac{(1-x)^2}{(1-\alpha)\rho_L} + \frac{x^2}{\alpha\rho_G}\right]$
Wadle (1999)	$P_2 - P_1 = \frac{1}{2}\cdot Z\cdot(1-\sigma^2)G_1^2\left[\frac{(1-x)^2}{\rho_L} + \frac{x^2}{\rho_G}\right]$

Bolle, 1997; Wadle, 1999; Aloui et al., 1999). Expressions for the pressure recovery have been developed by assuming the velocity of both phases are uniform and in thermal equilibrium. In most cases, the additional wall shear stresses in the developing region downstream of the expansion is neglected and the wall pressure at the downstream face of the expansion is assumed equal to the upstream pressure.

In this case, the pressure recovery is reduced to a function of the change in the two-phase flow momentum only. The existing models for the pressure recovery are summarized in Table 1. Attou et al. (1997) noted that the homogenous model such as Mendler (1966) tends to overestimate the experimental results due to the assumption of no slip between the phases. This assumes the liquid decelerates as much as the gas when it passes through the expansion, which leads to a higher prediction of the pressure recovery. On the other hand, models which assume no momentum transfer between the phases (Delhaye, 1981; Wadle, 1999) tend to underestimate the experimental results. In this case, the liquid is assumed to decelerate much less than the gas when it passes through the expansion due to its higher inertia. Consequently, the model predicts a smaller decrease in the momentum, which leads to a lower predicted pressure recovery. Several models have been developed based on the mechanical energy equation, including Lottes (1960), who ignored the gas mass flow rate and assumed that all losses take place in the liquid phase only, and Richardson (1958) who considered the two phases but assumed the same void fraction across the expansion. Mendler et al. (1961) used the single-phase equation with a two-phase correction factor.

There have been attempts to incorporate the flow regimes into the pressure recovery models to improve their accuracy. Schmidt and Friedel (1996) and Attou et al. (1997, 1999) argued that the momentum transport between the two phases, including the slip ratio and wall shear stress, should significantly depend on the flow regime. The effect of the flow regime has been introduced to the analysis through external closure relations (Attou and Bolle, 1999), while some models have been developed for specific flow regimes. For example, Attou et al. (1997, 1999) developed a model for bubbly flow, while Schmidt and Friedel (1996) developed a model for annular-mist flow across a sudden expansion.

The objective of the present work is to develop a more complete formulation of the pressure recovery of two-phase flow across a sudden expansion. Experiments were performed using air–oil to evaluate the relative contribution of the different terms in the formulation under different upstream flow conditions. The pressure recovery formulation is presented in the next section followed by a description of the test facilities. An analysis of the results are then presented, followed by the conclusions of this study.

2. Pressure recovery formulation

The mass and momentum conservation equations are applied to the control volume (V) bounded by the surface (A) as shown in Fig. 1, which is occupied by the phases k ($k = G, L$). The wall pressure at the downstream face of the expansion (P_w) is assumed lower than the pressure just upstream of the expansion section due to the energy losses in the recirculating zone. Subscripts 1 and 2^* are used to designate the flow conditions at the section slightly upstream of the expansion and a section where the flow is fully developed, respectively.

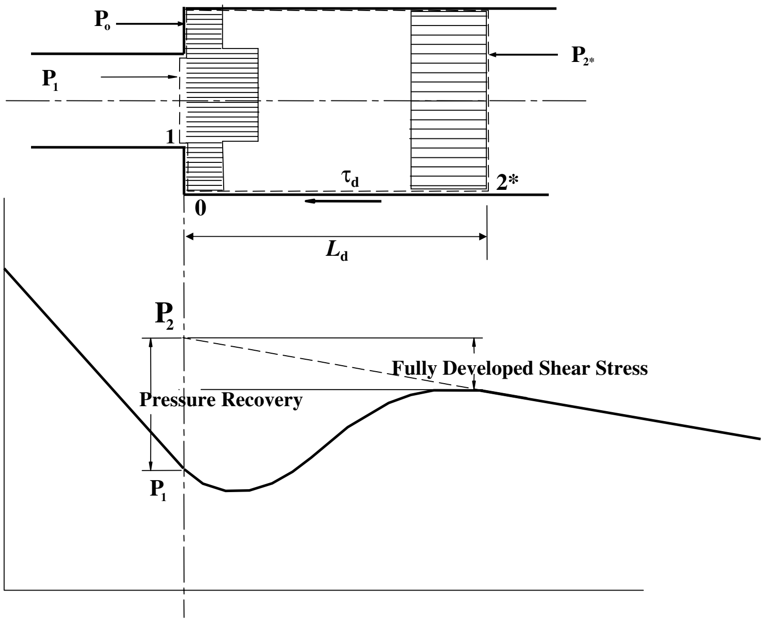


Fig. 1. Schematic of pressure distribution across sudden expansion.

The pressure recovery due to the sudden expansion is defined as the difference in the pressure when the fully developed pressure gradient lines upstream and downstream of the expansion are extrapolated to the point of the area change (Fig. 1). The following assumptions are used in the present analysis:

(1) Steady adiabatic two-phase flow.
(2) No mass transfer between the two phases.
(3) Uniform velocity of each phase at the inlet and outlet of the control volume.
(4) The body forces on both phases are neglected.
(5) Uniform pressure at any cross-section.
(6) The surface tension forces are neglected, therefore, the pressure of both phases are equal at any cross-section.

The conservation of mass for the gas and liquid is

$$\sum_{k,i}(-1)^i \int_{A_i} \overline{\rho_k v_{k,i}} \cdot \mathrm{d}A = 0 \tag{1}$$

for $k = G, L$ and $i = 1, 2$, where G and L refer to the gas and liquid phases.

With the above assumptions, Eq. (1) can be written as

$$\rho_L A_{L1} V_{L1} + \rho_G A_{G1} V_{G1} = \rho_L A_{L2} V_{L2} + \rho_G A_{G2} V_{G2} \tag{2}$$

where the subscripts 1 and 2 refer to upstream and downstream of the sudden expansion. The average velocity of gas and liquid is

$$\begin{aligned} V_{Gi} &= \frac{M_G}{\alpha_i A_i \rho_G} \\ V_{Li} &= \frac{M_L}{(1-\alpha_i) A_i \rho_L} \end{aligned} \tag{3}$$

where M and α are the mass flow rate and void fraction. The continuity equation for each phase is

$$V_{G1} A_{G1} \approx V_{G2} A_{G2} \tag{4}$$

$$V_{L1} A_{L1} \approx V_{L2} A_{L2} \tag{5}$$

By introducing an equivalent velocity V_0 that satisfies

$$M_{L1} = (1 - x)M_0 = \rho_L V_0 A_1 (1 - x) \tag{6}$$

where V_0 is the inlet velocity assuming that the entire flow is liquid and x the mass quality, the average velocities can be expressed as

$$V_{L1} = \frac{V_0(1 - x)}{(1 - \alpha_1)}, \quad V_{L2} = \frac{\sigma(1 - \alpha_1)V_{L1}}{(1 - \alpha_2)} \tag{7}$$

$$V_{G1} = \frac{\rho_L V_0(x)}{(\alpha_1)\rho_G}, \quad V_{G2} = \frac{\sigma\alpha_1 V_{G1}}{\alpha_2} \tag{8}$$

Here σ is the area ratio $\left(\frac{A_1}{A_2}\right)$. The axial momentum equation along the pipe axis (z) applied to the control volume shown in Fig. 1 is

$$\sum_{k,i}\left((-1)^i \int_{A_i} \overline{\rho_k v_{k,i}(v_{k,i})} \cdot \mathrm{d}A\right) = \sum_k \int_0^{L_d} \pi D_2 \overline{\tau_k} \cdot \mathrm{d}z - \sum_{k,i} \int_{A_i} \overline{p_{k,i}} \cdot \mathrm{d}A \tag{9}$$

The above can be written as

$$\sum_{k,i}(-1)^i \rho_k A_{i,k}(V_{k,i}^2) = \int_0^{L_d} \pi D \cdot \tau_d \cdot \mathrm{d}z + A_1 \cdot P_1 - A_2 \cdot P_2 + \int_{d/2}^{D/2} P_{\mathrm{w}}(r) \cdot 2\pi r \cdot \mathrm{d}r \tag{10}$$

Here τ_d is the wall shear stress in the developing region downstream of the sudden expansion, and d and D are the upstream and downstream pipe diameter. The average wall pressure, (P_0), is defined as

$$P_0 = \frac{\int_{d/2}^{D/2} P_{\mathrm{w}}(r) \cdot 2\pi r \cdot \mathrm{d}r}{(A_2 - A_1)} \tag{11}$$

Substituting the above into Eq. (10), the one-dimensional momentum equation can be written as

$$P_1 A_1 + P_0(A_2 - A_1) + \rho_L A_{L1} V_{L1}^2 + \rho_G A_{G1} V_{G1}^2 = P_{2^*} A_2 + \rho_L A_{L2} V_{L2}^2 + \rho_G A_{G2} V_{G2}^2 + \pi \int_0^{L_d} D_2 \tau_d(z) \cdot \mathrm{d}z \tag{12}$$

Since the flow is fully developed beyond L_d, the pressure downstream of the sudden expansion section for $z > L_d$ can be expressed as

$$(P_{2^*} - P(z)) \cdot A_2 = \pi D_2 \tau_{fd}(z - L_d) \tag{13}$$

where τ_{fd} is the wall shear stress in the fully developed region. The extrapolated value of the pressure at $z = 0$, $P(0) = P_2$, is given by

$$P_2 = P_{2^*} + \frac{4}{D_2}\tau_{fd} L_d \tag{14}$$

Substituting Eq. (3), (14) and the separated flow model presented by Eqs. (7) and (8) into Eq. (12) results in

$$P_2 - P_0 - \sigma(P_1 - P_0) = \rho_L V_0^2 \sigma \cdot \left[(1 - x)^2\left(\frac{1}{1 - \alpha_1} - \frac{\sigma}{1 - \alpha_2}\right) + (x^2)\frac{\rho_L}{\rho_G}\left(\frac{1}{\alpha_1} - \frac{\sigma}{\alpha_2}\right)\right] - \frac{4}{D_2}\left(\int_0^{L_d} \tau_d(z) \cdot \mathrm{d}z - \tau_{fd} L_d\right) \tag{15}$$

An explicit expression for the pressure recovery can be obtained from Eq. (15) as

$$P_2 - P_1 = \rho_L V_0^2 \sigma \cdot \left[(1 - x)^2\left(\frac{1}{1 - \alpha_1} - \frac{\sigma}{1 - \alpha_2}\right) + (x^2)\frac{\rho_L}{\rho_G}\left(\frac{1}{\alpha_1} - \frac{\sigma}{\alpha_2}\right)\right] - (P_1 - P_0)(1 - \sigma) - \frac{4}{D_2}\left(\int_0^{L_d} \tau_d(z) \cdot \mathrm{d}z - \tau_{fD} L_d\right) \tag{16}$$

The above takes into account the change in void fraction and the losses in the recirculating zones which results in a lower pressure on the downstream face of the expansion section, as well as the additional viscous wall shear stress in the developing region downstream of the expansion. The first term on the right hand side of Eq. (16) is identical to the Romie model and represents the change of the momentum flux of the two phases across the expansion. The second term takes into account the pressure difference between the upstream flow and the downstream face of the sudden expansion. This term reduces to zero at $\sigma = 1$, for which the singularity degenerates. The third term represents the additional wall shear stress in the developing flow downstream of the sudden expansion.

3. Experimental facilities

Experiments were performed in an air–oil two-phase flow loop shown schematically in Fig. 2 to evaluate the contributions of the different terms to the pressure recovery. The facility operates at pressures up to 415 kPa, with maximum air and oil flow rates of 43 kg/h and 1840 kg/h, respectively. Oil is supplied from a reservoir through a gear pump that is controlled through a programmable speed controller. Air is taken from the main laboratory air supply at approximately 4 bar and regulated to the desired pressure through a pressure regulator. The air and oil are then mixed through an annular mixer. In the mixer, the oil flows on the outside of an inner perforated pipe, while the air flows in the inner pipe and enters the oil stream through 380 holes of 0.79 mm diameter. The air–oil mixture then passes through the horizontal test section, which has a total length of 3.5 m. Two turbine flow meters in parallel, with ranges of 0.026–0.51 kg/s and 0.0036–0.255 kg/s, were used to measure the oil volumetric flow rate. Four calibrated rotameters in a bank were used to measure the air volume flow rate, with a range of 2×10^{-5} to 1.12×10^{-2} kg/s, and an accuracy of $\pm 1\%$ of full scale. The pressure and temperature of the air were measured at the inlet of the test section in order to determine the mass flow rate of air and then the mass quality.

System pressure was monitored at five different locations throughout the loop using Bourdon pressure gauges as shown in Fig. 2. The pressure distribution along the upstream and downstream sections were measured using two differential pressure transducers, one absolute pressure transducer and two 12 channel pressure switching units. The absolute pressure transducer was connected to the first pressure tap along the test section which was used as a reference, while the differential pressure transducers were used to measure the pressure difference between the first pressure tap and the other pressure taps distributed along the test section. The upstream pressure taps were connected to the higher pressure range differential pressure transducer

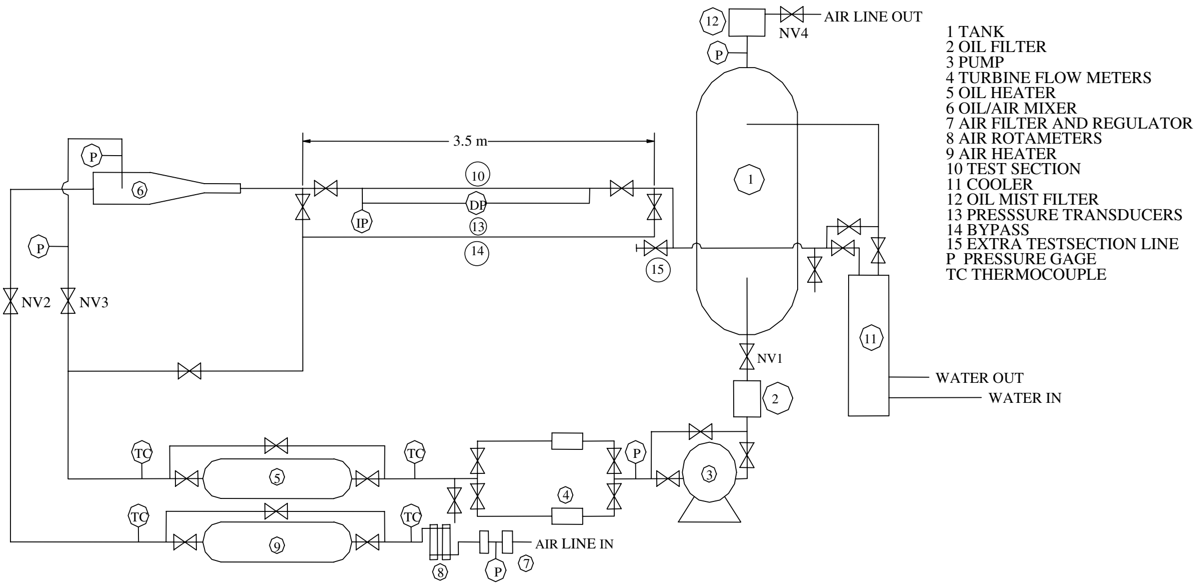


Fig. 2. Schematic of air–oil two-phase flow loop.

(0–248.6 kPa) with an accuracy of ±0.5%, while the downstream pressure taps were connected to the low pressure range differential pressure transducer (0–35 kPa) with an accuracy of ±0.25% full scale.

Three area expansion ratios of approximately 0.0625, 0.25 and 0.444 were used for this study. Commercially available clear polycarbonate tubing was used for both upstream and downstream sections to allow for flow visualization. The area ratios of 0.0625 and 0.25 were obtained using 6.35 and 12.7 mm (0.25 and 0.5 in.) diameter tubing for the upstream section with a length of 1.11 m, and 25.4 mm (1 in.) diameter tubing for the downstream section with a length of 2.5 m. The 0.444 area ratio was obtained by using 12.7 mm (0.5 in.) and 19.05 mm (0.75 in.) diameter tubing for the upstream and downstream sections. The pressure was measured through 1.6 mm diameter pressure taps drilled through the tube wall and carefully smoothed to ensure no burrs protruded into the flow area. Seven pressure taps were distributed upstream, with eight pressure taps on the downstream section as shown in Fig. 3. Four pressure taps were located on the downstream face of the sudden expansion wall as shown in Fig. 4. The pressure taps in the vicinity of the expansion

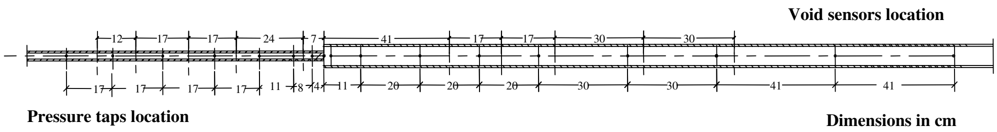


Fig. 3. Schematic diagram of test section (area ratio 0.0625) showing locations of pressure taps and void sensors.

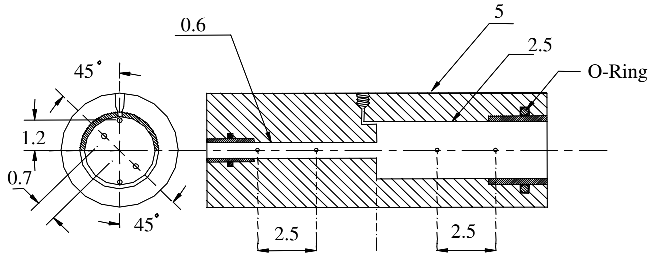


Fig. 4. Locations of pressure taps on the downstream face of the sudden expansion (dimensions in cm).

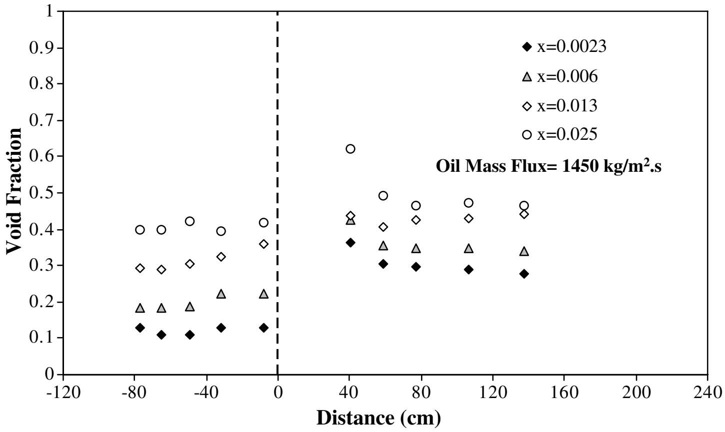
Table 2
Experimental test conditions

Mass quality	Area ratio	J_{L1} (m/s)	J_{G1} (m/s)	G_{L1} (kg/m^2 s)
0.003–0.47	0.0625	0.1–4.3	0.1–17.5	175–1500
0.0007–0.57	0.25	0.05–1.6	0.1–44.7	80–820
0.004–0.67	0.444	0.01–0.66	0.14–28	20–2050

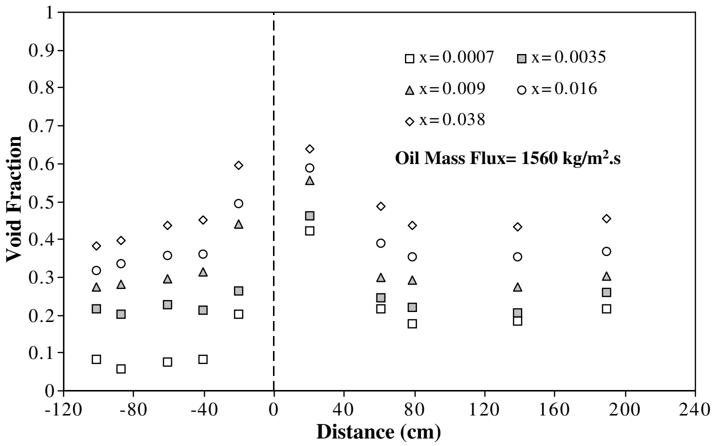
Table 3
Uncertainty in the measured and calculated quantities

Quantity	Absolute uncertainty	Percentage uncertainty
Temperature	±0.2 °C	
Liquid mass flux		±2–10%
Mass quality		±3–4.5%
Local void fraction		±4–7%
Void fraction (capacitance sensor)		±3–5%
Local liquid velocity		±5–7%
Local void fraction		±5%
Predicted average void fraction		±4–8%
Measured pressure recovery		±4–6%
Predicted pressure recovery		±11–19%

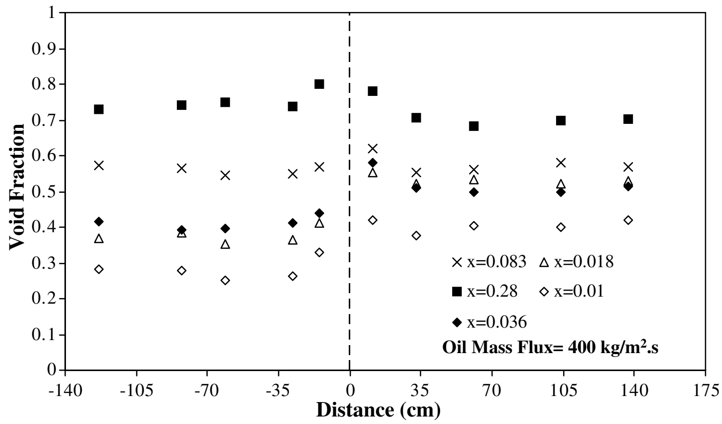
section were placed closer, while further from the sudden expansion the taps were placed farther apart. Flow regimes were identified by visual observation with the aid of a 1000 frames/s high-speed video camera. The cross-sectional average void fraction at different locations along the test section were obtained using



(a) $\sigma = 0.0625$



(b) $\sigma = 0.25$



(c) $\sigma = 0.444$

Fig. 5. Distribution of void fraction across the sudden expansion.

capacitance sensors located along the test section as described by Ahmed et al. (2004). The measurement volume of the capacitance sensor (length = $1.65D$) is small compared to the pipe length, and the void fraction can be assumed to represent the average void fraction at a section along the pipe.

Experiments were performed for the flow conditions listed in Table 2, where J is the superficial velocity and G is the mass flux. The upstream flow regimes corresponding to these flow conditions were bubbly, intermittent and annular-mist, while the fully developed downstream flow regimes were bubbly, stratified wavy, intermittent and annular-mist. The uncertainties of the measured and calculated quantities are summarized in Table 3. A number of tests were carried out with single-phase flow using both oil and air to validate the experimental loop and verify the instrumentation and experimental methodology. These measurements also provide a limit for the two-phase flow data as the mass quality approaches zero and one. The axial pressure distribution data for both the single-phase oil and single phase air were in good agreement with the theoretical pressure gradient for different Reynolds numbers.

4. Results and discussion

4.1. Void fraction distribution

Representative time-averaged void fraction distributions along the test section for different oil mass flux and mass quality for the three area ratios are presented in Fig. 5. In all cases, the void fraction increases as the flow approaches the sudden expansion, and this phenomenon is most prominent at the highest oil mass flux (Fig. 5b). As the two-phase flow approaches the sudden expansion, it slows down under the influence of the adverse pressure gradient immediately downstream of the sudden expansion. The gas phase is more affected by the adverse pressure gradient due to its lower axial momentum, and slows down more significantly than the liquid phase causing the increase in void fraction. There is a sharp increase in the void fraction just downstream of the sudden expansion, followed by a gradual decrease to approximately a constant value. The sudden increase in void fraction immediately downstream of the sudden expansion is likely due to gas recirculation in the vicinity of the sudden expansion as observed in the flow visualizations, with the gas phase occupying most of the recirculation zone immediately downstream of the expansion. This is plausible, as the liquid with a higher inertia will tend to follow the main streamline flow direction, rather than the recirculatory streamlines of the gas phase. For example, for the area ratio of 0.0625 and oil mass flux of 1450 kg/m^2 s and quality of 0.0023, the void fraction increases from a value of about 0.11 far upstream to about 0.36 at the closest downstream measurement location (40.6 cm) to the sudden expansion section (Fig. 5a), with the relaxation complete to a void fraction of about 0.28 at about 76 cm (30D) downstream of the expansion.

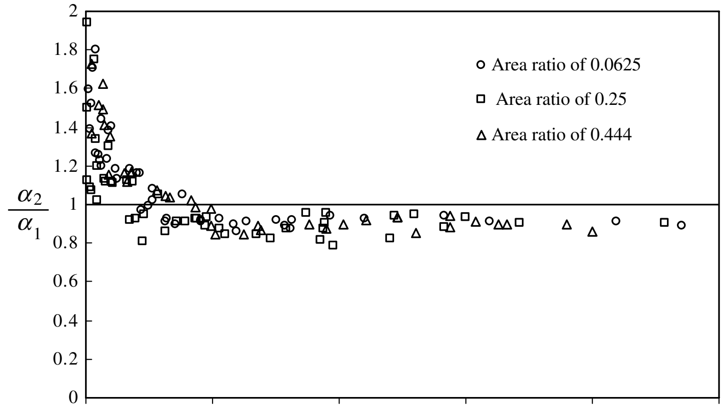
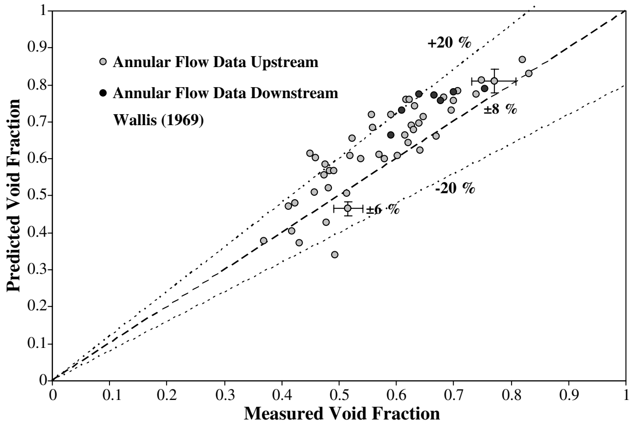
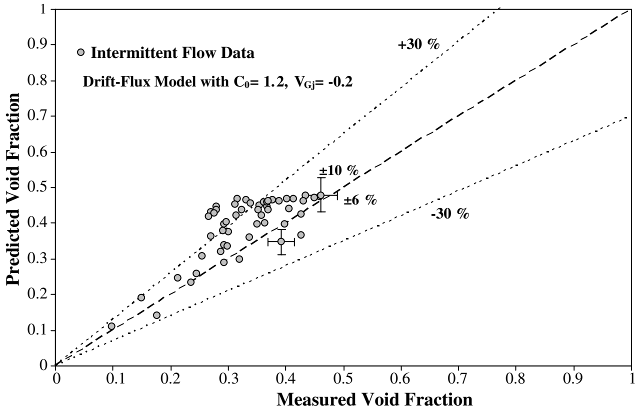


Fig. 6. Variation of change of void fraction across sudden expansion with mass quality.

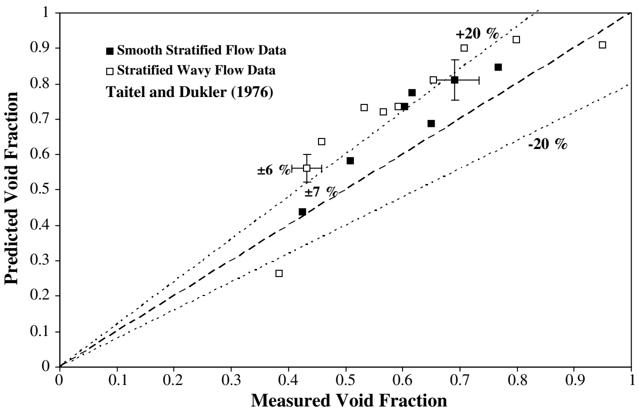
The fully developed void fraction downstream of the sudden expansion can either decrease or increase from its upstream value due to the change in the pipe diameter, which can cause a change in the flow pattern downstream of the expansion. For the area ratio of 0.25 and mass quality of 0.0007, the fully developed void fraction downstream is higher than the upstream value (Fig. 5b). In this case, the flow pattern changed from elongated bubble upstream to intermittent (slug) flow downstream of the sudden area expansion. The



(a) Wallis (1969) model



(b) Franca and Lahey (1992) model

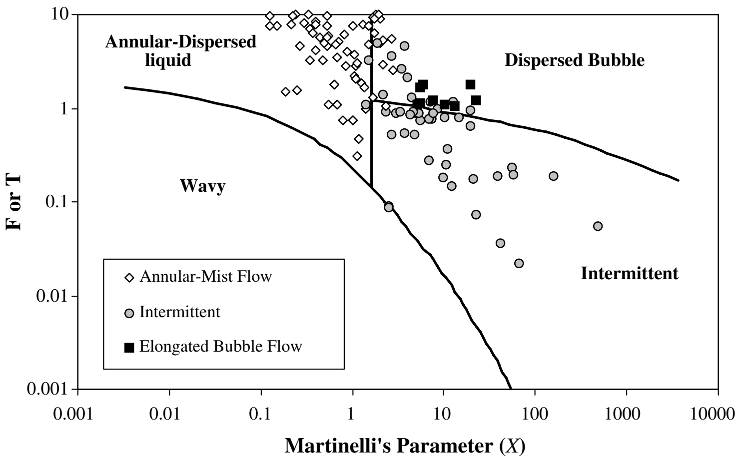


(c) Taitel and Dukler (1976) model

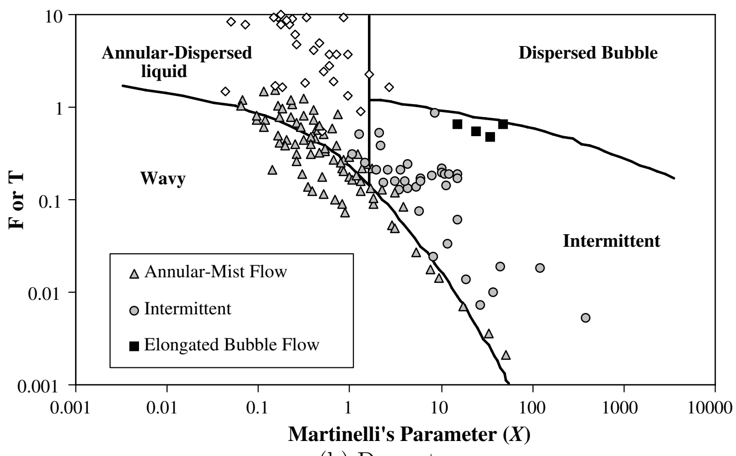
Fig. 7. Comparison of the fully developed void fraction with model predictions.

difference between the upstream and downstream void fraction becomes smaller as the mass quality increases to 0.038 where intermittent flows are observed both upstream and downstream of the sudden expansion. For the area ratio of 0.0625, the fully developed void fraction downstream of the sudden expansion remains higher than the upstream value for mass qualities less than 0.025 (Fig. 5a). On the other hand, for area ratio of 0.444 the downstream void fraction is higher at the low mass qualities and becomes approximately the same at a mass quality equal to 0.083 (Fig. 5c). As the mass quality is further increased, the fully developed downstream void fraction becomes smaller than the fully developed void fraction upstream of the sudden expansion. The change in void fraction for all the area ratios considered here is plotted as a function of the mass quality in Fig. 6. At low void fractions, corresponding to mass quality less than 0.05, the downstream void fraction is higher than the upstream void fraction. As the mass quality increases (and the flow regime becomes annular upstream), the downstream void fraction tends to decrease from its upstream value by approximately a constant value of 10% over a wide range of mass quality. The downstream flow regime in this case was stratified, where the liquid level initially increases due to the adverse pressure gradient, before it reaches an approximately constant value in the fully developed region.

Existing models based on the flow regime to predict the fully developed void fraction in a horizontal pipe were compared to the current fully developed upstream and downstream experimental data. For annular flow, the two-cylinder model proposed by Wallis (1969) was able to predict the experimental results to within ±20%



(a) Upstream



(b) Downstream

Fig. 8. Comparison of upstream and downstream flow regime data with the Taitel and Dukler (1976) flow map.

as shown in Fig. 7a. The drift-flux model with the distribution parameters presented by França and Lahey (1992) for slug flow predicted the void fraction for most of the intermittent flow patterns to within ±30%. The model tends to be in fairly good agreement with the experimental data for void fraction less than 0.3 where the flow pattern is approximately a slug flow, while the prediction accuracy decreases as the void fraction increases as shown in Fig. 7b. However, this trend was expected since the distribution parameters in this model are for slug flow and cannot give good results for other intermittent flow patterns. The experimental results for stratified flow were compared to the mechanistic model developed by Taitel and Dukler (1976), and agreed to within ±20%. As expected, the model tends to predict the smooth stratified flow better than the stratified wavy flow data as shown in Fig. 7c.

The fully developed flow regimes upstream and downstream of the sudden expansions were in good agreement with the flow map of Taitel and Dukler (1976) over the entire range of the present flow conditions as shown in Fig. 8. In this figure, F and T are

$$F = \sqrt{\frac{\rho_G}{\rho_L - \rho_G}}\frac{J_G}{\sqrt{D \cdot g}}$$

$$T = \left[\frac{\left(\frac{dP}{dx}\right)_L^S}{(\rho_L - \rho_G) \cdot g}\right]^{1/2}$$

and the Martinelli parameter X is defined as

$$X = \left(\frac{1-x}{x}\right)^{0.9}\left(\frac{\rho_G}{\rho_L}\right)^{0.5}\left(\frac{\mu_L}{\mu_G}\right)^{0.1}$$

Given also that the fully developed void fractions were predicted through the conventional models using the pipe geometry and the flow conditions suggested that once the flow becomes fully developed downstream of

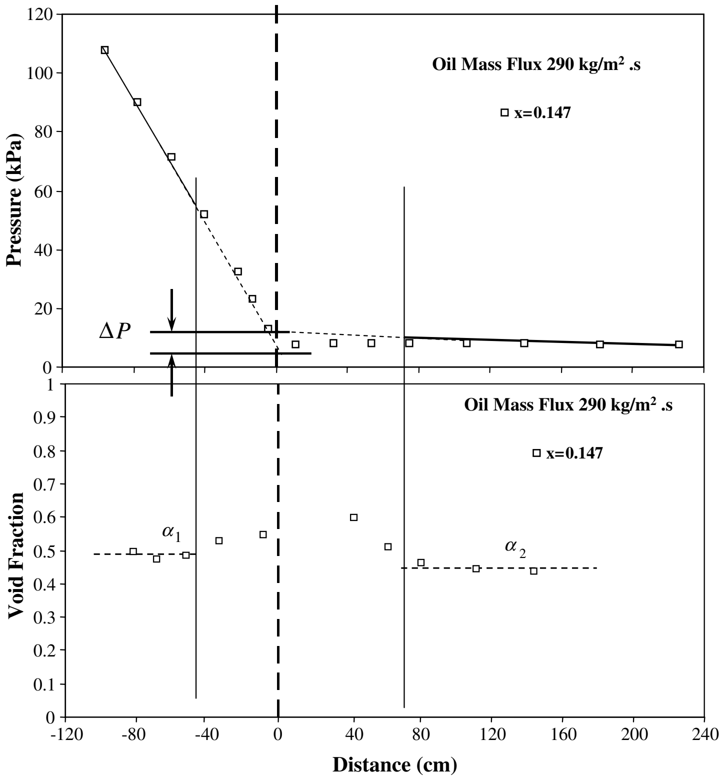


Fig. 9. Illustration for determination of pressure recovery.

the expansion, the flow has lost its ‘memory’ on how it was formed. However, the two-phase flow parameters in the developing region and the length of the developing region is strongly dependent on the area ratio and the upstream flow regime.

4.2. Pressure distribution and recovery

The pressure recovery is evaluated from the fully developed pressure distributions upstream and downstream of the sudden expansion. The axial location where the flow becomes fully developed downstream of the sudden expansion is dependent on the area ratio and the upstream flow parameters, such as flow regime, mass quality and phase velocities. In most previous studies, the fully developed region in the two-phase flow

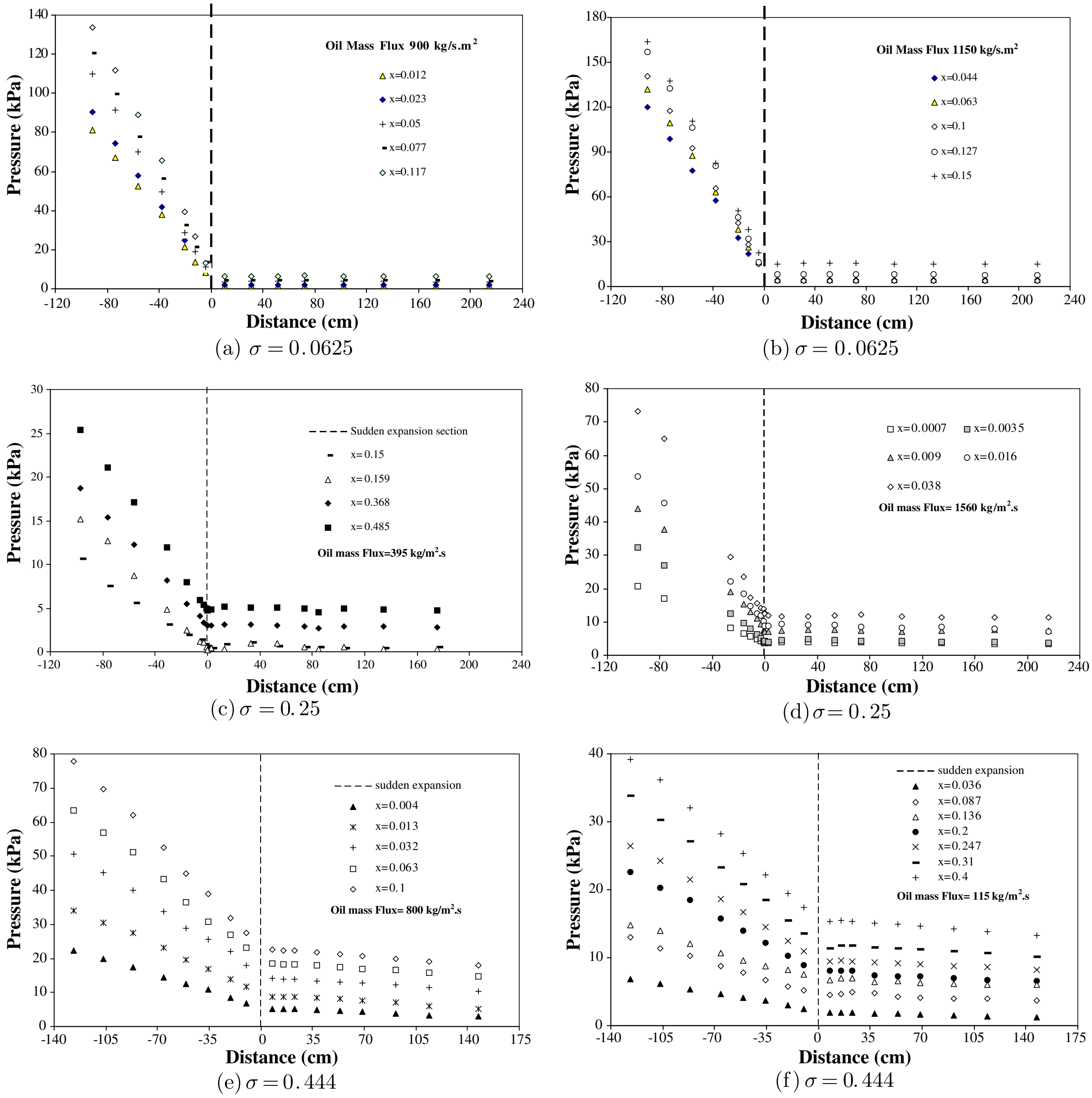


Fig. 10. Static pressure distribution across the sudden expansion.

was selected by examining the pressure distribution. The choice in identifying the region where the two-phase flow becomes fully developed can affect the slope of the pressure gradient and hence the value of the pressure recovery. In the present study, the flow is considered fully developed in the regions where the mean void fraction is nearly constant, and the pressure gradient lines are fitted to the data in these regions. This method is illustrated in Fig. 9 for a representative test condition of oil mass flux of 290 kg/m^2 s and mass quality of 0.147 for area ratio of 0.25. The pressure recovery is estimated from the values obtained from the extrapolation of these lines to the point of the sudden expansion.

Representative pressure distribution data for the three area ratios and for different oil mass flux and mass quality are presented in Fig. 10. As expected, the pressure gradient upstream of the expansion increases as the quality increases because the liquid velocity increases with a consequent increase in the friction losses. The pressure is relatively constant for a short distance after the expansion, followed by a rise due to the area increase and deceleration of both phases. This is also reflected in the void fraction profile, where the void fraction increases rapidly as the gas phase decelerates more than the liquid phase downstream of the sudden expansion section as discussed earlier. The pressure then decreases due to pipe friction, with the pressure gradient in the downstream section significantly smaller than the upstream section due to the larger cross-sectional area.

Pressure measurements at different locations on the downstream face of the sudden expansion were obtained for area ratios of 0.0625 and 0.4444 to determine its difference from the centerline of the sudden expansion. The pressure at the centerline of the expansion, P_1, was obtained by extrapolating the fully developed upstream pressure gradient line to the sudden expansion section. The ratio between the local wall pressure and the extrapolated upstream pressure, P_w/P_1, is consistently less than one and varies as a function of the mass quality, oil mass flux and area ratio as shown in Fig. 11. The lowest values of P_w/P_1 were associated with annular flow in the upstream section. For example, for the area ratio of 0.0625 and oil mass flux of 900 kg/m^2 s, as the mass quality increased from 0.012 to 0.077, P_w/P_1 decreased from 0.91 to 0.57 and increased again to 0.64 as the mass quality further increased to 0.12 (Fig. 11a). A similar trend was observed for a mass flux of 1150 kg/m^2 s (Fig. 11b). The ratio P_w/P_1 is higher (approximately 0.8) for area ratio of 0.444

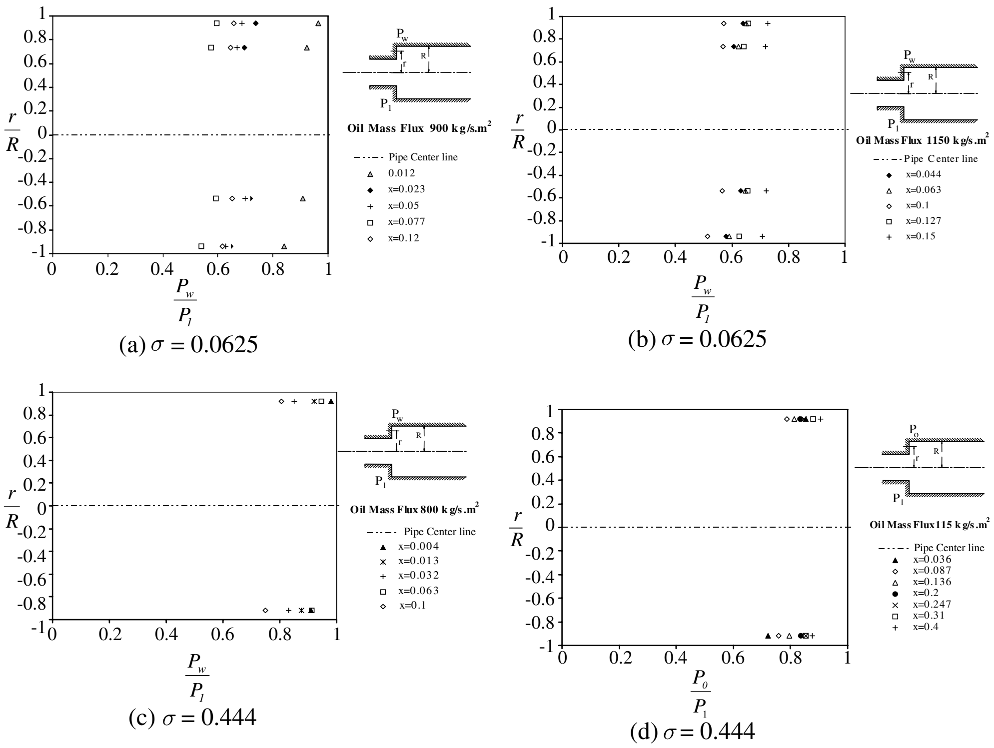


Fig. 11. Pressure distribution on downstream face of sudden expansion.

as shown in Fig. 11c and d. The pressure on the top wall is approximately 3–6% higher than on the bottom wall for the different flow conditions. The lower pressure in the bottom wall of the pipe is likely due to the higher losses in the bottom recirculation zone where there is a higher liquid content than in the upper zone.

The difference between the centerline and average wall pressure normalized by the pressure recovery is plotted against the quality in Fig. 12. For both area ratios, the value increases from a very small value at low mass

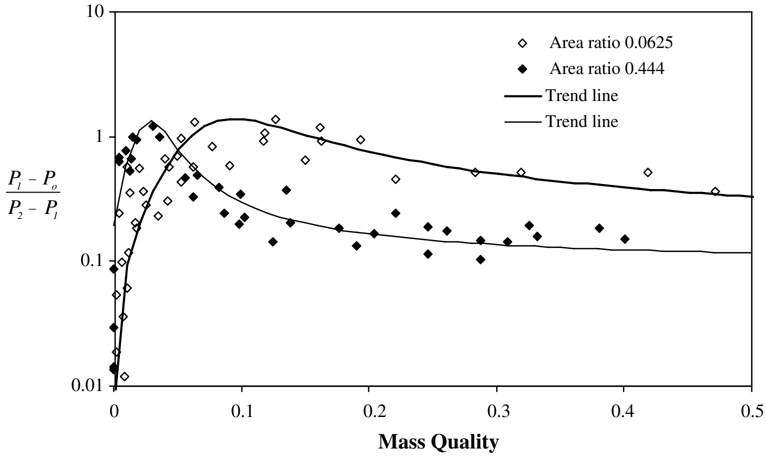


Fig. 12. Variation of wall pressure on downstream face of the expansion with mass quality.

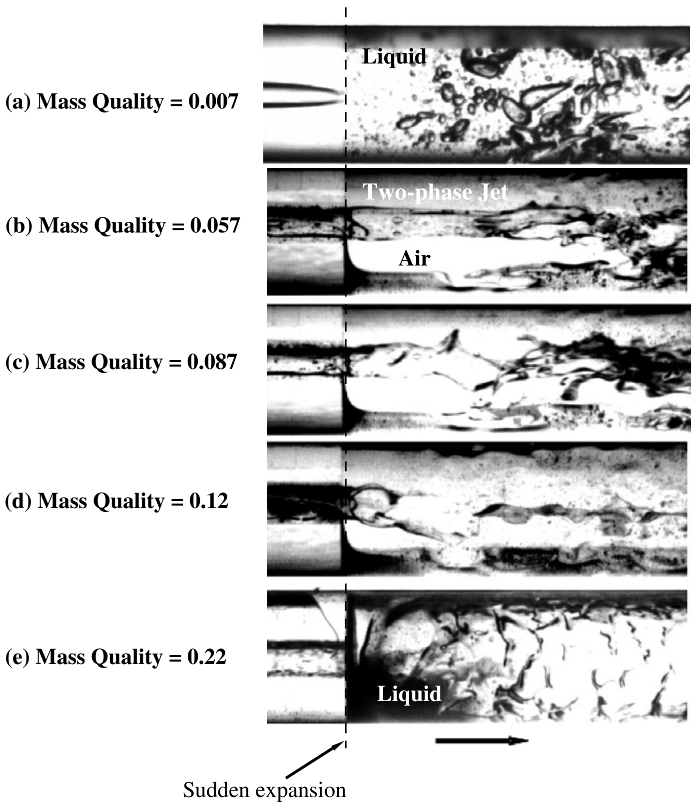


Fig. 13. Flow pattern change immediately downstream of the sudden expansion with increasing mass quality (area ratio 0.0625 and $G_{L1} = 1050\ \mathrm{kg/m^2\ s}$).

quality to a maximum value which is of the same order as the pressure recovery, close to the region where the upstream flow becomes annular, and then decreases with further increase in quality. For the area ratio of 0.0625, the maximum change in the wall pressure from the upstream pressure occurs at mass quality of 0.08 and decreases to about 50% of the pressure recovery in the range of mass quality from 0.2 to 0.5. The same trend was found for the area ratio of 0.444. However, in this case, the maximum change occurs at a lower mass quality of 0.04 and then decreases to about 20% of the pressure recovery in the range of mass quality from 0.1 to 0.5. For a given mass quality, the change of the average wall pressure from the upstream pressure is higher for the area ratio of 0.0625 than for area ratio of 0.444 as shown in Fig. 12. This is mainly due to the higher losses in the circulation zone in the smaller area ratio. These results show that the assumption of a constant pressure at the downstream face of the expansion equal to the upstream pressure is valid only for single phase flow or at a very low mass quality.

The variation of the wall pressure change with mass quality can be explained with the aid of the flow visualization images shown in Fig. 13 for area ratio of 0.0625. At a mass quality of 0.007, the liquid phase is dominant in the region immediately downstream of the expansion (Fig. 13a). As the mass quality increased, there is a separation of the two-phase jet just downstream of the expansion, which creates a lower pressure region at the downstream face of the expansion (Fig. 13b–d). As the mass quality was further increased and the upstream flow becomes annular, there is no distinct two-phase jet (Fig. 13e) and P_0/P_1 begin to increase. A

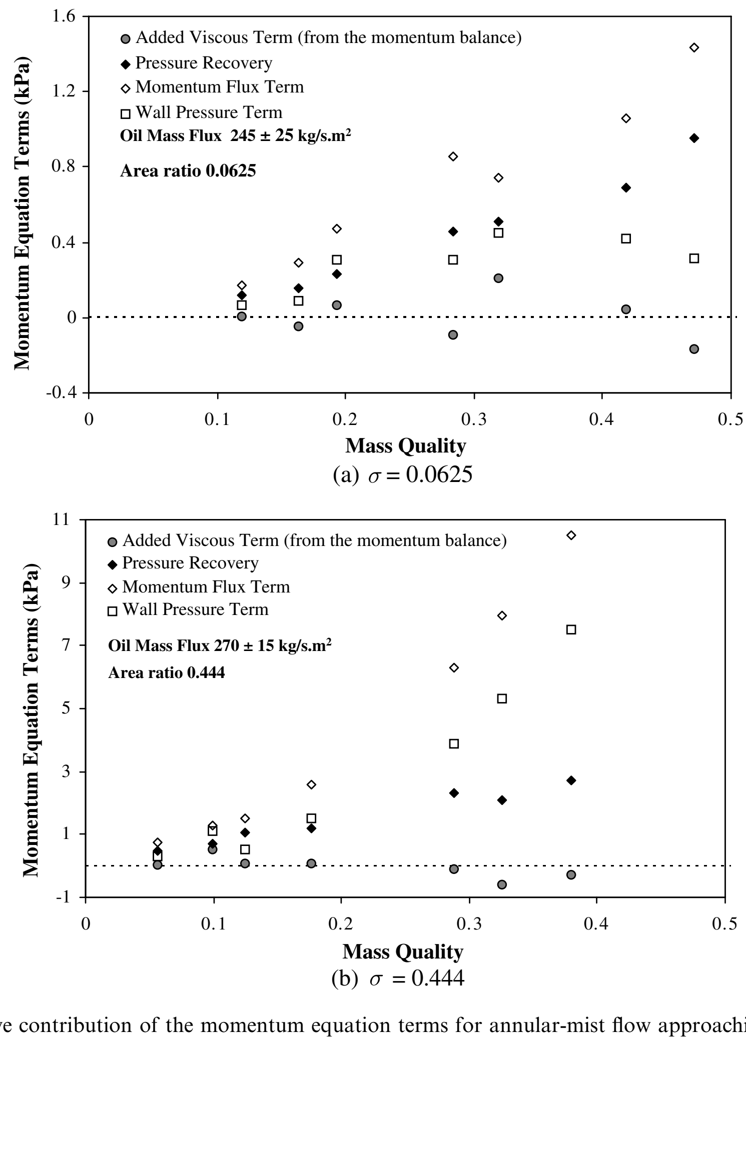


Fig. 14. Relative contribution of the momentum equation terms for annular-mist flow approaching the sudden expansion.

further increase of the mass quality results in a further increase in P_0/P_1 and approaches 1 as the mass quality approaches 1 for single phase air flow.

5. Evaluation of the pressure recovery formulation

The pressure recovery given by Eq. (16) is expressed as a function of the change in the momentum of the two phases, the pressure difference between the upstream flow and the downstream face of the expansion, and the additional wall shear stress in the developing region. Using the experimental data for the upstream and downstream void fraction, wall pressure on the downstream face and the mass quality, the first two terms can be evaluated. Hence, from the measured pressure recovery, the additional wall shear stress term can be estimated using Eq. (16). The relative contributions of these terms to the pressure recovery are shown in Fig. 14 for annular flow upstream of the sudden expansion. The data are for approximately constant liquid mass flux. For both area ratios, the contribution of the wall pressure term on the downstream face is more significant than the additional wall shear stress. The significance of the wall pressure term increases as the mass quality increases. As the annular two-phase flow passes through the sudden expansion, the air tends to separate from the main stream towards the recirculation zone downstream of the expansion (Fig. 13b). In this case, the additional wall shear stress is mainly due to the air in the recirculation zone and is negligibly small.

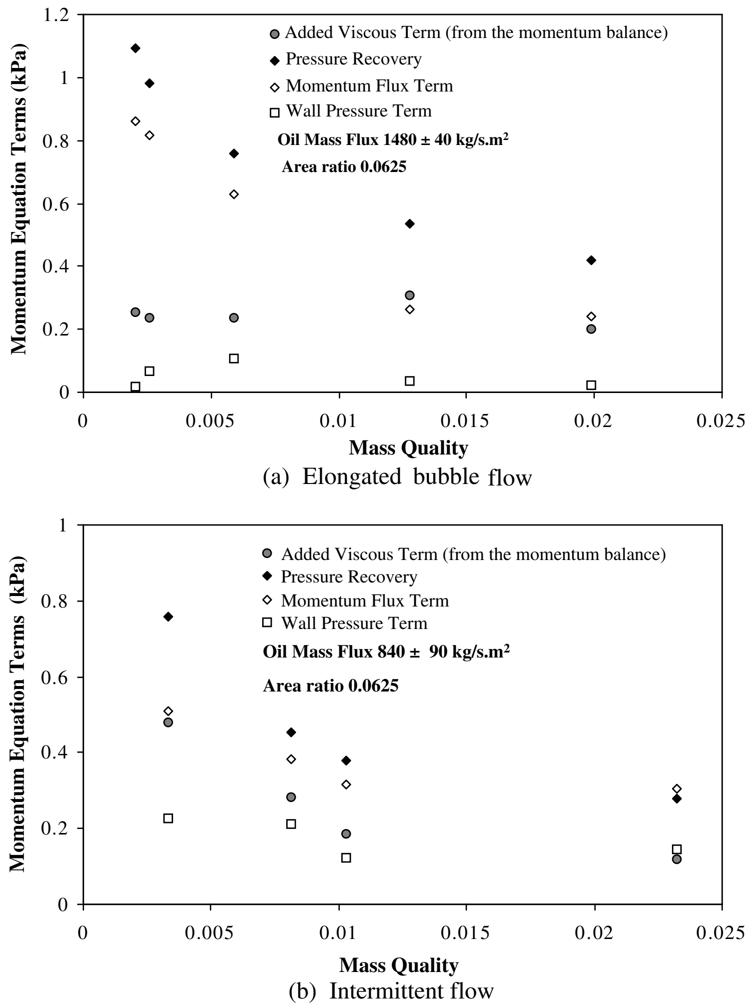


Fig. 15. Relative contribution of the momentum equation terms for intermittent two-phase flow approaching the sudden expansion.

However, the wall pressure on the downstream face can be considerably smaller than the pressure just upstream of the expansion. Accordingly, in modeling the pressure recovery in the case of annular flow upstream of the sudden area expansion, the effect of wall pressure should be accounted for while the additional wall shear stress in the developing region is negligible.

On the other hand, for elongated bubbly and intermittent flows approaching the sudden expansion, the additional wall shear stress is more significant than the wall pressure on the downstream face as shown in Fig. 15. In this case, the liquid phase is more dominant in the recirculation zone as shown in the flow visualization of Fig. 13a. This agrees with the results of Aloui et al. (1999) for bubbly flow in a vertical sudden area expansion. For the higher liquid mass flux (1480 kg/m^2 s) as shown in Fig. 15a, the additional wall shear stress

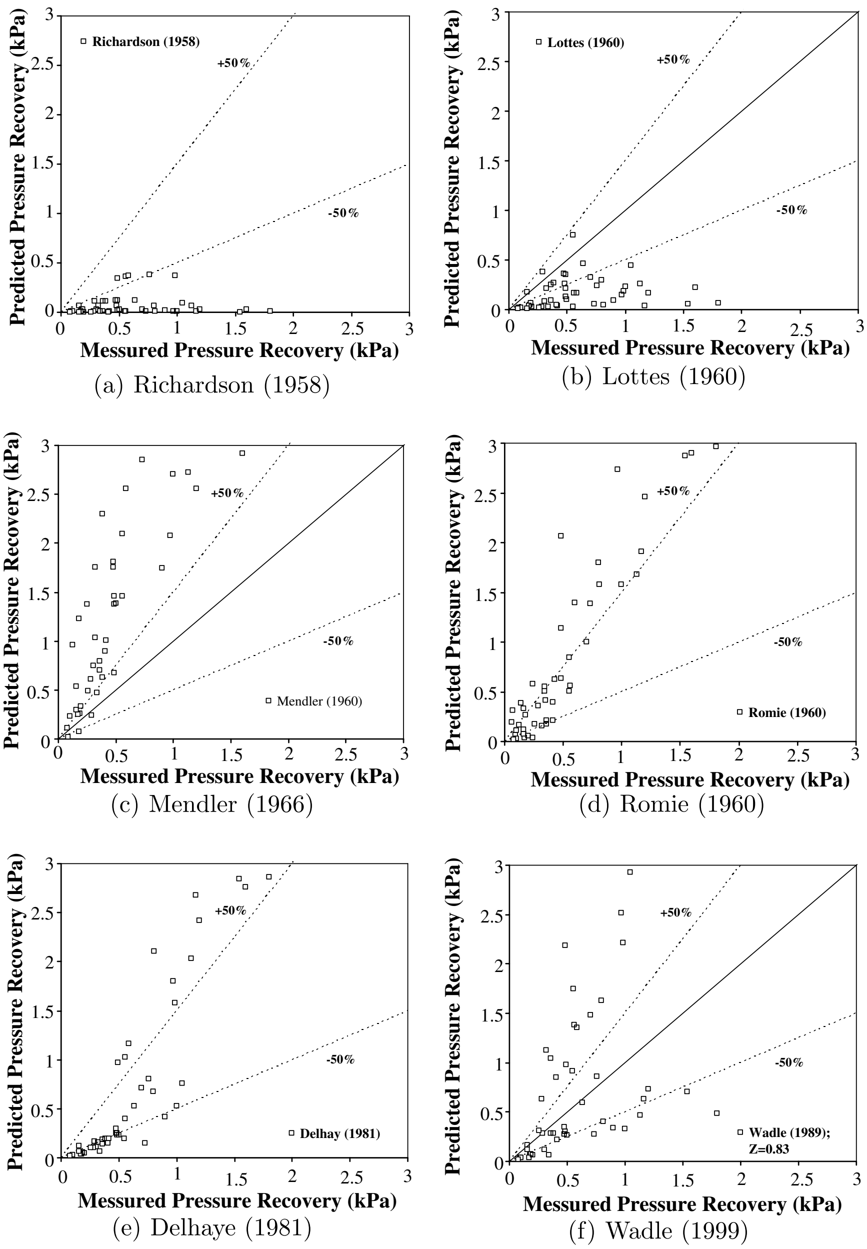


Fig. 16. Comparison between experimental results and existing pressure recovery models.

term is nearly constant over the mass quality range, while it decreases as the mass quality increases for lower mass flux (840 kg/m^2 s) as shown in Fig. 15b.

The measured pressure recovery values are compared with the values predicted from the existing models (Table 1) in Fig. 16 and the root mean square errors are summarized in Table 4. The measured values of the void fraction and the wall pressure were used in the calculations of the pressure recovery. In general, the prediction from models based on the momentum equation (Delhaye, 1981) are in better agreement with the experimental data than those from the models based on the mechanical energy equation (Lottes, 1960). This is because the mechanical energy equation does not take into account the dissipative terms in the control volume explicitly. The models based on the mechanical energy equation, including Lottes (1960), who ignored the gas mass flow rate, and Richardson (1958) who considered the same void fraction across the expansion, were significantly lower than the experimental values. The model proposed by Mendler et al. (1961) which used the energy equation for single-phase flow with a two-phase correction factor tends to significantly over estimate the experimental data. On the other hand, the models based on the momentum equation by Delhaye (1981) and Romie (Lottes, 1960) tend to better predict the experimental data. The Romie model takes into account the void fraction change across the expansion, which improves the prediction of the pressure recovery over Delhaye (1981). The empirical correlation proposed by Wadle (1999) had approximately the same root mean square error as Romie's model, but with a much larger relative standard deviation. The formulation proposed by Romie is found to be closest to the experimental data, with an average root mean square error of about 47% over the entire range of mass qualities.

The generality of the present formulation (Eq. (16)), including the void fraction change and wall pressure correlations, was evaluated by comparing other existing data to the present model predictions. The experimen-

Table 4
Relative error of model predictions

Model	Root mean square (%)	Relative standard deviation (%)
Romie model	47.6	52.8
Lottes (1960)	77.4	21.2
Richardson (1958)	91	14.3
Mendler et al. (1961)	63.2	67.8
Delhaye (1981)	54.3	87.5
Wadle (1999)	48.3	101.6

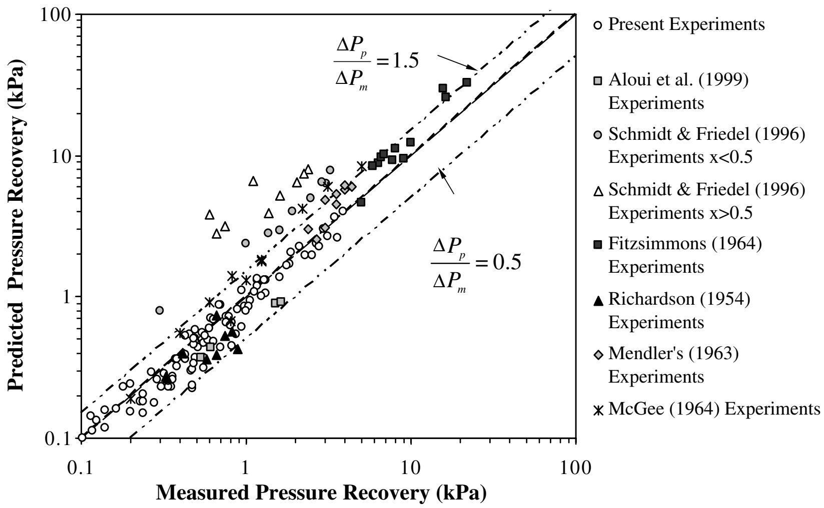


Fig. 17. Comparison of existing experimental pressure recovery data with the predicted values using the present formulation.

Table 5
Relative error of pressure recovery data with present formulation

Mass quality	Root mean square (%)	Relative standard deviation (%)	n
$x \geqslant 0.5$	380	50	10
$x < 0.5$	34	25	182

tal data for two component two-phase flow through horizontal sudden expansions by Richardson (1958), Schmidt and Friedel (1996) and Aloui et al. (1999) were compared against the present model predictions. In addition, the steam water data of Fitzsimmons (1964), Mendler et al. (1961) and McGee (1966), were also used for comparison. The previous studies for steam water two-phase flow did not report any change in the mass quality across the sudden expansion. For the model predictions, the upstream and downstream flow patterns were identified using the flow pattern maps of Taitel and Dukler (1976) and Hewitt and Roberts (1969) for vertical flows. The void fraction was estimated using the appropriate correlations for both the upstream and downstream void fraction based on the flow pattern. The wall pressure term was obtained by extrapolating the present data (Fig. 12) to different area ratios and higher mass qualities, while the additional wall shear stress term was estimated using the correlation of Aloui et al. (1999) for bubbly flow.

The existing data along with the present data are compared against the predicted values using the present formulation in Fig. 17. Most of the data are in a good agreement with the predicted values to within ±35%, with the exception being the data of Schmidt and Friedel (1996) at mass qualities higher than 0.5 as listed in Table 5. This is likely because the error involved in estimating the wall pressure term at mass qualities higher than 0.5 by extrapolating the current data becomes more significant. The inclusion of the wall pressure term improved the prediction of the pressure recovery for mass qualities lower than 0.5.

6. Conclusions

The pressure recovery of two-phase flow across a horizontal sudden area expansion was formulated using a one-dimensional separated flow model. In this formulation, the change in the void fraction across the sudden expansion, the additional wall viscous shear stress in the developing region and the pressure difference between the upstream flow and the downstream face of the expansion were taken into account. Experiments were performed using air–oil two-phase flow to evaluate the relative contribution of the different terms to the pressure recovery.

In the fully developed regions, both the upstream and downstream flow regimes were well predicted by the flow pattern map of Taitel and Dukler (1976). In addition, the fully developed void fractions were in good agreement with existing void fraction models. This indicates that once the flow is fully developed downstream of the expansion, it has no memory of its formation. However, the flow in the developing region and the developing length are dependent on the upstream flow pattern and the area ratio.

At a very low mass quality, where intermittent flow patterns were observed upstream of the sudden expansion, the pressure difference between the upstream flow and the downstream face of the expansion was very small. This difference increased when the upstream flow pattern was annular. The contribution of the wall pressure on the downstream face of the expansion is more significant than the additional viscous shear stress when the upstream flow is annular. On the other hand, for intermittent flows, the wall pressure term tends to be very small compared to the additional viscous shear stress. Including the wall pressure and the additional viscous shear stress terms in the pressure recovery formulation improved the prediction of the pressure recovery to better than ±30%.

Acknowledgements

The support of the Natural Sciences and Engineering Research Council (NSERC) of Canada and Pratt & Whitney Canada is gratefully acknowledged.

References

Ahmed, W.H., Ching, C.Y., Shoukri, M., 2004. Capacitance sensors for measurement of void fraction and flow pattern identification in gas–liquid flows. In: Proceedings CSME Forum 2004, CD-ROM.

Aloui, F., Doubliez, L., Legarand, J., Souhar, M., 1999. Bubbly flow in an axisymmetric sudden expansion: pressure drop, void fraction, wall shear stress, bubble velocities and sizes. Exp. Therm. Fluid Sci. 18, 118–130.

Attou, A., Bolle, L., 1997. Integral formulation of balance equations for two-phase flow through a sudden enlargement. part 1: basic approach. Proc. Insti. Mech. Eng., Part C: J. Mech. Eng. Sci. 211, 387–397.

Attou, A., Bolle, L., 1999. Theoretical analysis of the two-phase steady flow characteristics parameters of a sudden enlargement. Z. Angew. Math. Phys. 50, 731–758.

Attou, A., Giot, M., Seynhaeve, M., 1997. Modeling of steady-state two-phase bubbly flow through a sudden enlargement. Int. J. Heat Mass Transfer 40, 3375–3385.

Delhaye, J.M., 1981. Singular pressure drops. In: Bergles, A.E. (Ed.), Two-phase and Heat Transfer in the Power and Process Industries. Hemisphere, Washington, DC.

Fitzsimmons, D.E., 1964. Two-phase pressure drops in piping components. Report of Work Performed Under Contract (45-1)-1350 between the Atomic Energy Commission and General Electric Company, HW-80970 REV1,Washington.

França, F., Lahey Jr., R.T., 1992. The use of drift-flux techniques for the analysis of horizontal two-phase flows. Int. J. Multiphase Flow 18, 787–801.

Hewitt, G.F., Roberts, D.N., 1969. Studies of two-phase flow patterns by simultaneous X-ray and flash photography. Technical report, AERE-M2159, UKAEA, Harwell.

Janssen, E., Kervinen, J.A., 1966. Two-phase pressure drop across contractions and expansions of water–steam mixture at 600 to 1400 psia. In Report Geap 4622-1965-US.

Lottes, P.A., 1960. Expansion losses in two-phase flow. Nucl. Sci. Eng. 9, 26–31.

McGee, J.W., 1966. Two-phase flow through abrupt expansion and contraction. Ph.D. thesis, North Carolina State University at Raleigh.

Mendler, O.J., 1966. Sudden expansion losses in single and two-phase flow. Ph.D. thesis, University of Pittsburgh, Pennsylvania.

Mendler, O.J., Rathbun, A.S., Van Huff, N.E., Weiss, A., 1961. Natural-circulation tests with water at 800 to 2000 psia under non-boiling, local boiling, and bulk boiling conditions. J. Heat Transfer 83, 261–273.

Richardson, B.L., 1958. Some problems in horizontal two-phase two-component flow. Ph.D. thesis, Purdue University, Indiana, USA.

Schmidt, J., Friedel, L., 1996. Two-phase pressure change across sudden expansions in duct areas. Chem. Eng. Commun. 141, 175–190.

Taitel, Y., Dukler, A.E., 1976. A model for predicting flow regime transitions in horizontal and near horizontal gas–liquid flow. Am. Inst. Chem. Eng. J. 22, 47–55.

Wadle, M., 1999. A new formula for the pressure recovery in an abrupt diffuser. Int. J. Multiphase Flow 15, 241–256.

Wallis, G.B., 1969. One-dimensional two-phase flow. McGraw-Hill, New York.